**CSE230: Discrete Mathematics**  
Practice Sheet 5: **Number Theory**

| Q1 | Determine whether 4 ∣ 7 and whether 4 ∣ 24 |
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| Q2 | What are the quotient and remainder when  a) 193 is divided by 7?  b) −42 is divided by 5?  c) 597 is divided by 23?  d) 30201 is divided by 33?  e) 0 is divided by 18?  f ) 3 is divided by 8? |
| Q3 | What are the possible remainders (according to the Division Algorithm) when an integer is  a) Divided by 4?  b) Divided by 9? |
| Q4 | For each of the following, find the quotient and remainder (guaranteed by the Division Algorithm)   1. When 17 is divided by 3. 2. When -17 is divided by 3. 3. When 73 is divided by 7. 4. When -73 is divided by 7. 5. When 436 is divided by 27. 6. When 539 is divided by 110. |
| Q5 | Today is Tuesday. Your uncle will come after 45 days. On which day (of the week) your uncle will be coming? |
| Q6 | Find the value of (177 **mod** 31 ⋅270 **mod** 31)**mod** 31 |
| Q7 | Show that if *a* ∣ *b* and *b* ∣ *a*, where *a* and *b* are integers, then *a* = *b* or *a* = −*b*. |
| Q8 | Prove or disprove that if *a* | *bc*, where *a*, *b* and *c* are positive integers and *a* ≠ 0, then *a* | *b* or *a* | *c*. |
| Q9 | Prove that if *a* is an integer that is not divisible by 3, then (*a* + 1)(*a* + 2) is divisible by 3. |
| Q10 | Let *m* be a positive integer. Show that *a* **mod** *m* = *b* **mod** *m* if *a* ≡ *b* (mod *m*). |
| Q11 | Considering *a* and *b* are integers, and  *a* ≡ *b* + 7 (**mod** 19), show that (*a*2 – 11) ≡ (*b*2 – 5*b*) (**mod** 19) |
| Q12 | If we consider 50 ≡ 23 (mod 9), then prove that 34 ≡ 16 (mod 9)  [Hint: Use the Theorem 5] |
| Q13 | 28. Find *a* **div** *m* and *a* **mod** *m* when  a) *a* = −991, *m* = 99.  b) *a* = −119, *m* = 101.  c) *a* = 10299, *m* = 999.  d) *a* = 12346, *m* = 101 |
| Q14 | Find the integer *a* such that  a) *a* ≡ 17 (mod 29) and −14 ≤ *a* ≤ 14.  b) *a* ≡ −11 (mod 21) and 90 ≤ *a* ≤ 110.  c) *a* ≡ 24 (mod 31) and −15 ≤ *a* ≤ 15.  d) *a* ≡ 99 (mod 41) and 100 ≤ *a* ≤ 140. |
| Q15 | Use modular exponentiation to find:  i) 7^64 **mod** 645.  ii) 32^203 **mod** 99.  iii) 22^3219 **mod** 243. |
| Q16 | i) What is the octal and hexadecimal expansion of (11011000110 01011)2.  ii) What is the hexadecimal expansion of (125110)10.  iii)What is the decimal expansion of (7716)8.  iv) What is the binary expansion of (BADDAD)16.  v) What is the octal expansion of (12344321)10. |
| Q17 | Expand the decimal number (506070)10 to the base ***x***, where ***x*** = ((10\*22)2 + 72) **mod** 23. |
| Q18 | If (*ab*)4 is a base-4 integer and (*ba*)7 is a base-7 integer, what is the largest possible value of (*a*+*b*)10 and why? Find non-zero values for a and b such that (*ab*)4=(*ba*)7, or prove that there are none. |
| Q19 | Determine whether the integers in each of these sets are **pairwise relatively prime**:  i) 14, 17, 85.  ii) 21, 34, 55.  iii) 25, 41, 49, 64.  iv) 17, 18, 19, 23. |
| Q20 | Consider two positive integers 5271 and 48714. Calculate the Greatest Common Divisor for the integers using Euclidean Algorithm. Also show the Least Common Multiple. |
| Q21 | Find the greatest common divisor of the following pair of numbers using the **Euclidean Algorithm**:  i) 11111, 111111.  ii) 1529, 14038.  iii) 750, 900.  iv) 414, 662. |
| Q22 | How many divisions are required to find gcd(21, 34) using the **Euclidean algorithm**? |
| Q23 | Find gcd(92928, 123552) and lcm(92928, 123552), and verify that gcd(92928, 123552) ⋅ lcm(92928, 123552) = 92928 ⋅ 123552 |
| Q24 | If the product of two integers is 2^7 · 3^8 · 5^2 · 7^11 and their greatest common divisor is 23345, what is their least common multiple? |
| Q25 | Determine whether the integers in each of these sets are pairwise relatively prime.  a) 11, 15, 19  b) 14, 15, 21  c) 12, 17, 31, 37  d) 7, 8, 9, 11 |
| Q26 | If *p* is a prime number, *p* > 3, then show that *p*2 ≡ *p*4 ≡ 1 (mod 3) |
| Q27 | Work out the GCD of 7105, 3185 and 2898 **only using the Euclidean Algorithm**.  Note that, gcd(*a*, *b*, *c*) = gcd(gcd(*a*, *b*), *c*) |
| Q28 | Given that, the square root of a number ***n*** when divided by 11 gives a remainder of 6 where **6 < √*n* < 28**. Find the number ***n***.Then, using modular exponentiation, calculate:  ***n* 125 *mod* 27**. |